Phys 231 Exam 2
March 2, 2018

NAME: Solutions

Student ID No.________________

Keep in mind that I must be able to follow your work in order to give you credit. This is especially true when awarding partial credit. A wrong answer with the right methods gets partial credit. If I cannot figure out the methods then no credit.

Answers must be justified. An answer by itself receives no credit.

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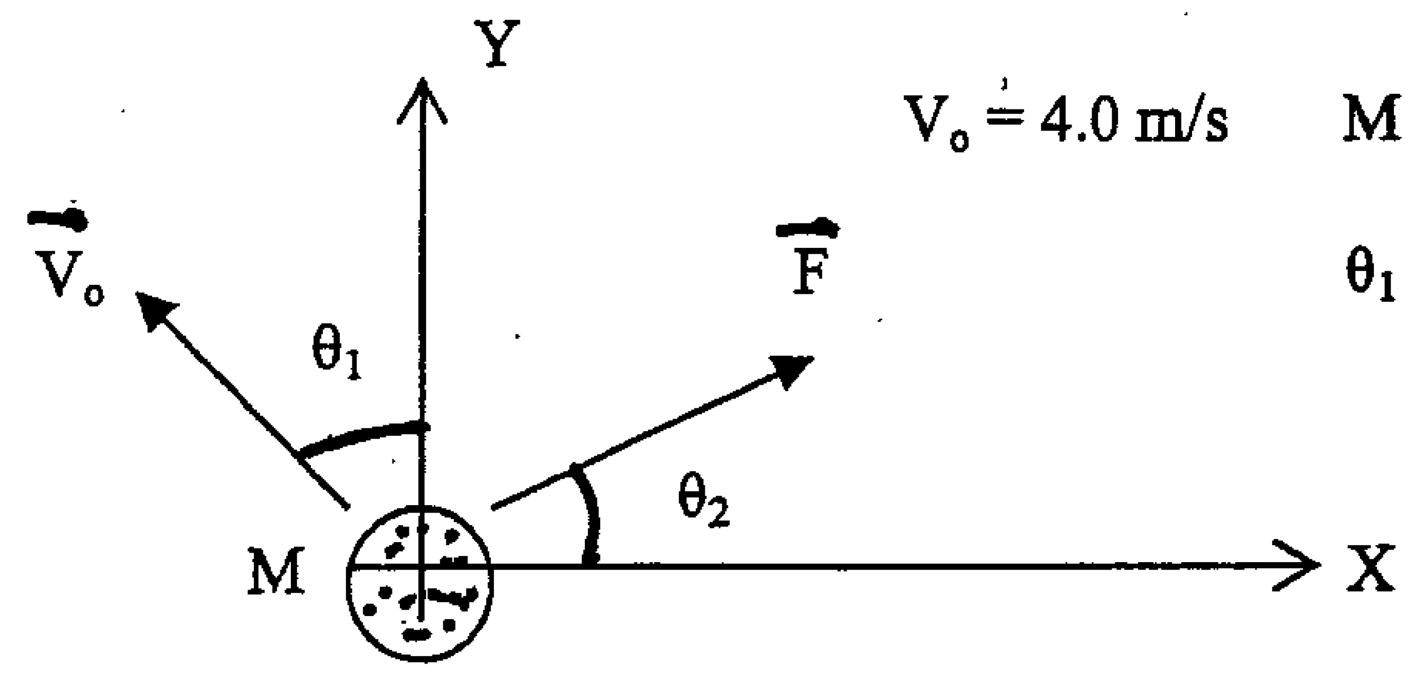
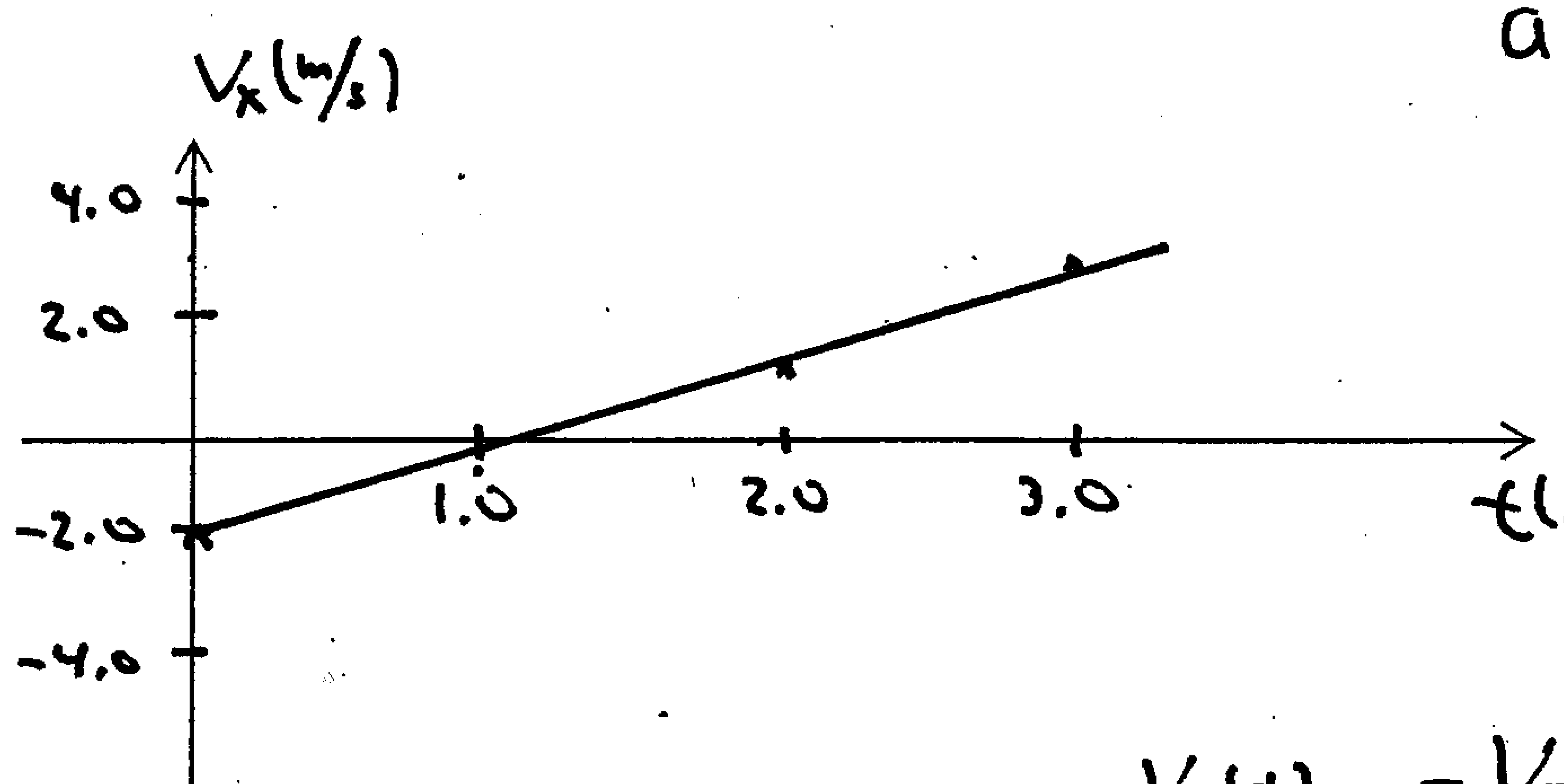
1) (15 pts) A mass M, initially located at the origin of the co-ordinate system, is free to slide on a frictionless table with an initial velocity V_o. A force F acts on the mass as depicted below and the initial velocity vector is given. Gravity and the normal force sum to zero.



$V_o = 4.0$ m/s $M = 1.5$ kg $F = 5.0$ N

$\theta_1 = 30°$ $\theta_2 = 60°$

a) In the space below, sketch a graph of the X–component of velocity (vertical axis) versus time (horizontal axis). Be certain to include labels, units, and a numerical scale on the axes of your graph. Include the first three seconds.



$$a_x = \frac{F\cos\theta_2}{m} = \frac{(5.0\,N)(\cos 60°)}{1.5\,kg} = +1.67\,m/s^2$$

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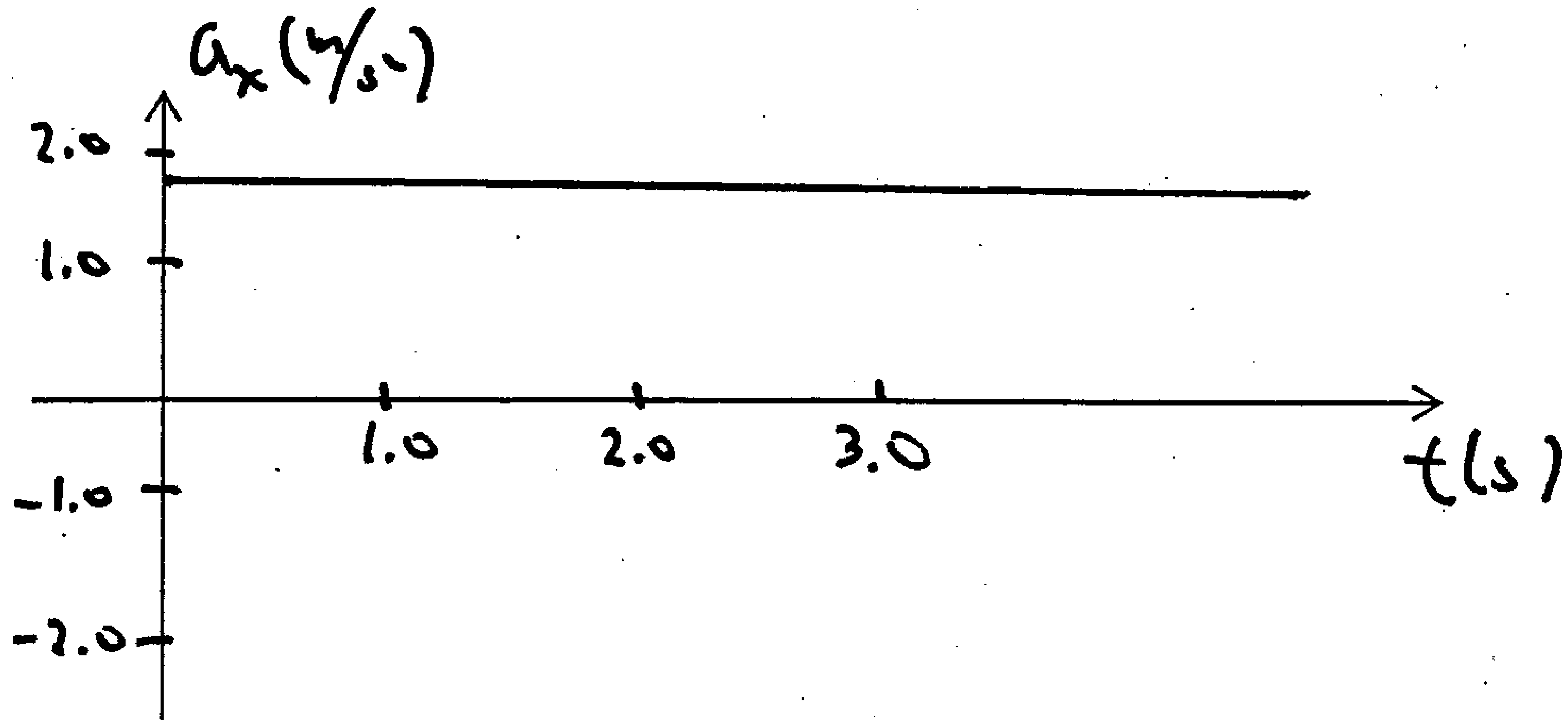
$$V_x(t) = -V_o \sin\theta_1 + a_x t = -2.0\,\tfrac{m}{s} + (1.67\,\tfrac{m}{s^2})\,t$$

b) In the space below, sketch a graph of the X-component of acceleration A_x (vertical axis) versus time (horizontal axis). Be certain to include labels, units, and a numerical scale on the axes of your graph. Include the first three seconds.



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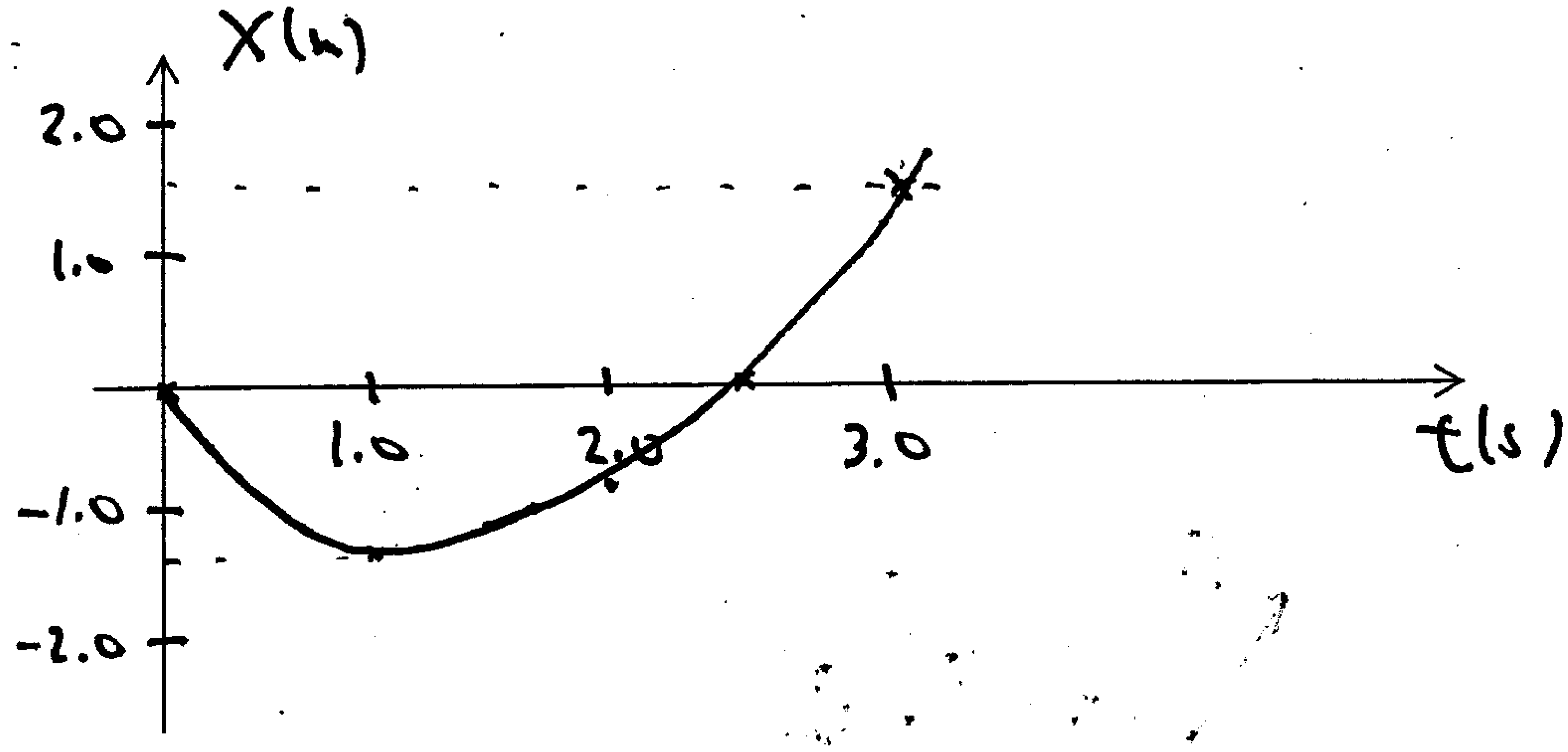
c) In the space below, sketch a graph of the X co-ordinate (vertical axis) versus time (horizontal axis). Be certain to include labels, units, and a numerical scale on the axes of your graph. Include the first three seconds.



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$$X(t) = X_0 - (V_0 \sin \theta_1)\,t + \tfrac{1}{2} a_x t^2$$

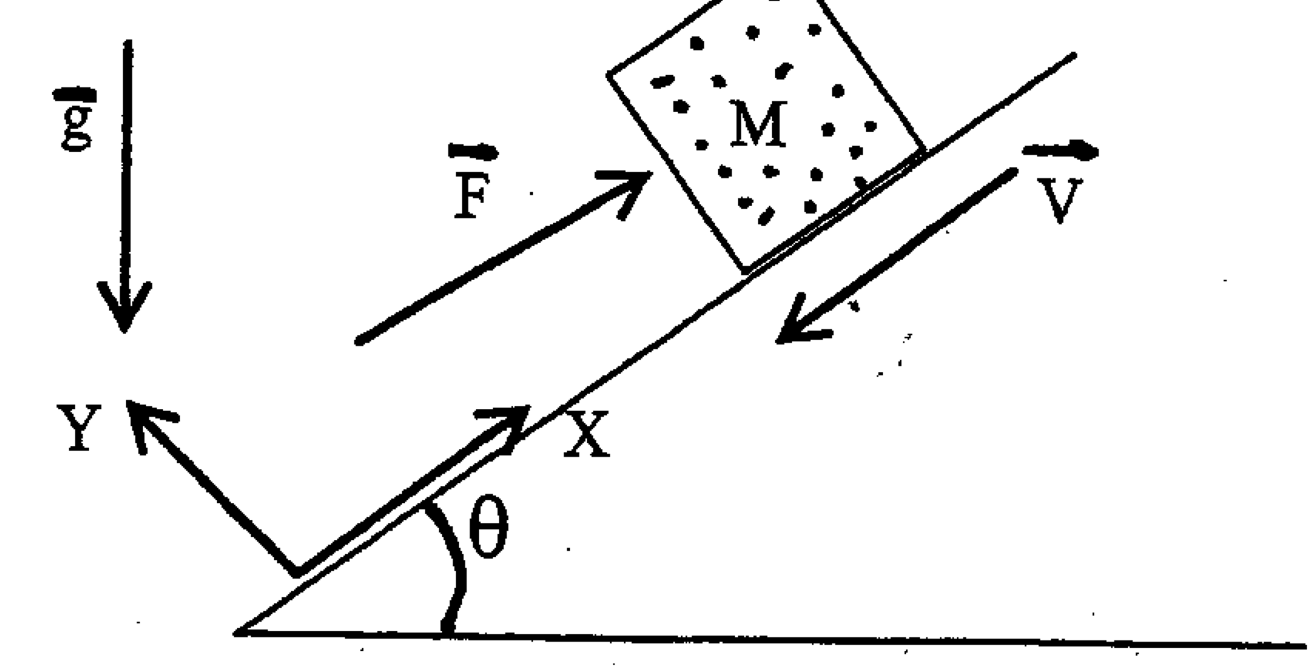
$$X_0 = 0 \qquad V_0 \sin \theta_1 = 2.0 \tfrac{m}{s} \qquad a_x = 1.67\ m/s^2$$

$$X(t) = (-2.0 \tfrac{m}{s})\,t + \tfrac{1}{2}(1.67 \tfrac{m}{s^2})\,t^2$$

Note: $X = 0$ when $t = \dfrac{2 V_0 \sin \theta_1}{a_x}$

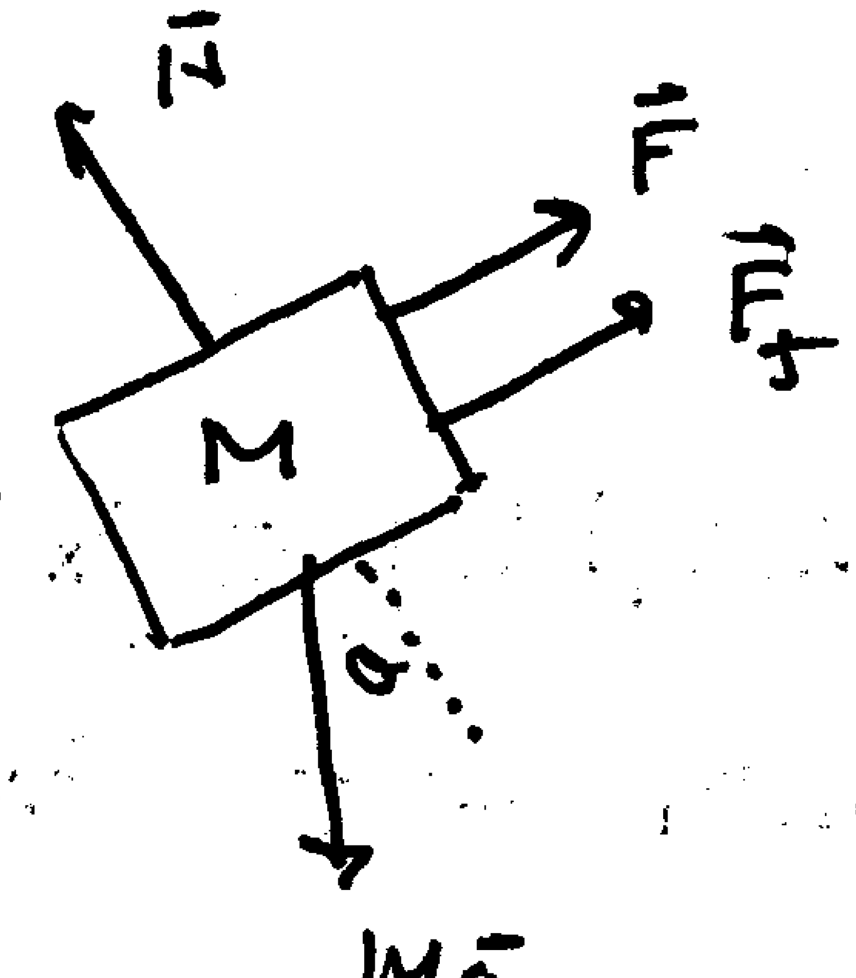
$$= 2.40\,s$$

2) (15 pts) The picture below shows a mass on an inclined plane. An external force F is applied as shown directly up the plane while the velocity of the mass is directed down the plane. The kinetic coefficient of friction is μ_k. Use the X-Y coordinate system shown on the diagram.



$M = 1.5$ kg
$\mu_k = 0.40$
$V = 5.0$ m/s
$F = 8.0$ N
$\theta = 30^0$

a) In the space below, draw a force diagram for the mass M.



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b) One of the forces in your force diagram should be a normal force. Calculate the magnitude of the normal force

$$N - Mg\cos\theta = 0$$
$$N = Mg\cos\theta$$
$$= (1.5\,kg)(9.8\,\tfrac{m}{s^2})(\cos 30^\circ)$$
$$= 12.7\,N$$

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c). Calculate the acceleration of the mass. Acceleration is a vector; give its direction.

$$F_f + F - mg\sin\theta = ma$$

$$F_f = \mu_k N = \mu_k mg\cos\theta$$

$$a = \mu_k g\cos\theta + \frac{F}{m} - g\sin\theta$$

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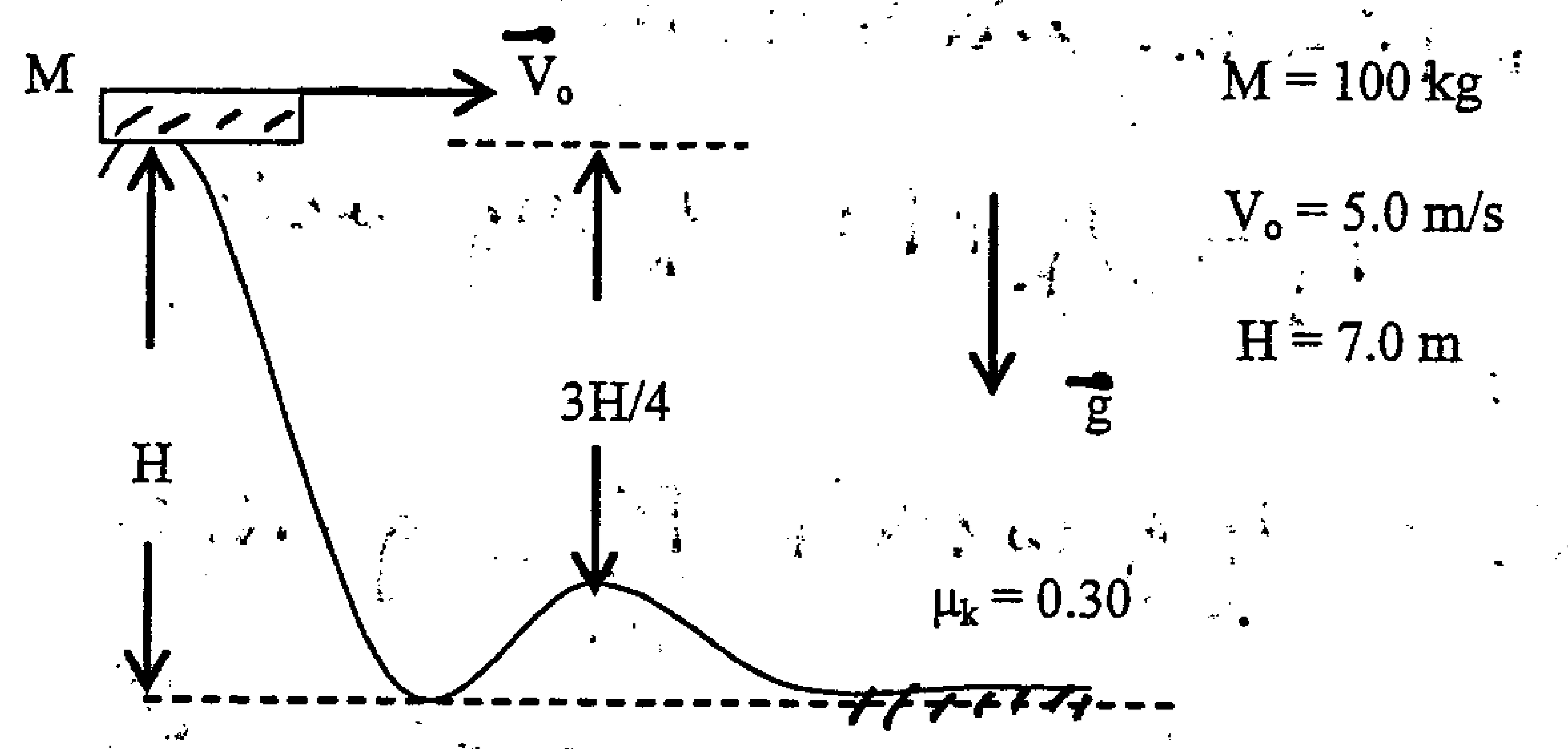
$$= g(\mu_k\cos\theta - \sin\theta) + \frac{F}{M}$$

$$= (9.8\tfrac{m}{s^2})[(0.4)(\cos 30^\circ) - \sin 30^\circ] + \frac{8.0\,N}{1.5\,kg}$$

$$= 3.82\ m/s^2 \text{ up the plane}$$

3) (20 pts) The picture below shows a point-like toboggan of mass M at the top of a hill with an initial speed of V_o. There is no friction along the toboggan run but there is friction at the bottom of the second hill where the surface becomes rough. This surface has a coefficient of kinetic friction $\mu_k = 0.30$



a) Calculate the work done by the force of gravity as the toboggan travels from the top of the first hill to the top of the second hill.

$$W = mg\left(\frac{3H}{4}\right)$$
$$= (100\ \mathrm{kg})(9.8\ \mathrm{m/s^2})\left(\frac{3}{4}\right)(7.0\ \mathrm{m})$$
$$= 5.1 \times 10^3\ \mathrm{J}$$

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c) What is the speed of the toboggan at the top of the second hill?

$$mgH + \frac{1}{2}mV_o^2 = mg\frac{H}{4} + \frac{1}{2}mV^2$$
$$V = \left(V_o^2 + \frac{3gH}{2}\right)^{1/2}$$
$$= \left[(5.0\ \tfrac{\mathrm{m}}{\mathrm{s}})^2 + \left(\tfrac{3}{2}\right)(9.8\ \tfrac{\mathrm{m}}{\mathrm{s^2}})(7.0\ \mathrm{m})\right]^{1/2}$$
$$= 11.3\ \mathrm{m/s}$$

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c) What is the work done by the force of friction as the toboggan comes to halt?

$$W_f = \Delta E_{mech} = E_f - E_i = -E_i \quad \text{since } E_f = 0$$

$$= -\left(mgH + \frac{1}{2} m V_0^2\right)$$

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$$= -(100\ kg)\left[(9.8\ \frac{m}{s^2})(7.0\ m) + \frac{1}{2}(5.0\ \frac{m}{s})^2\right]$$

$$= -8.1 \times 10^3\ J$$

d) How far does the toboggan slide along the surface with friction before coming to a halt?

$$W_f = -F_f x \qquad F_f = \mu_k N = \mu_k mg$$

$$W_f = -\mu_k mg x$$

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$$x = -\frac{W_f}{\mu_k mg} = -\frac{(-8.1 \times 10^3\ J)}{(0.30)(100\ kg)(9.8\ \frac{m}{s^2})}$$

$$= 27.6\ m$$